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Infrasonic Far-Field Radiation From a Turbulent Wake

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13. ABSTRACT

The acoustic far-field pressure and intensity due to a turbulent wake is calculated using Lighthill's free-turbulence theory. The radiation intensity is plotted as a function of the Mach number and of the ratio of wake wavelength to radiating wavelength.

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INFRASONIC FAR-FIELD RADIATION FROM A TURBULENT WAKE

INTRODUCTION

The problem considered in this short report is the calculation of the far-field intensity (or pressure) due to radiation from volume turbulence. The specific question to be answered is: What is this far-field intensity at extremely low frequencies (infrasound)?

In the second section the far-field pressure is calculated using Lighthill's relation for the pressure due to volume turbulence (1,2). To do this requires certain assumptions about the turbulent velocity field, namely, that it be isotropic and have the form of an outgoing spherical wave. The basic result of the second section is that the infrasonic farfield intensity is proportional to $M^4(\Lambda/\lambda)^6$ where M is the Mach number, Λ is the turbulent wavelength, and λ is the radiated wavelength. For infrasound (large λ) this factor is very small. A plot is given showing the intensity as a function of M and Λ/λ.

Finally, the last section contains a short summary of both assumptions and omissions.

THEORY

The spectral pressure $p(\mathbf{r})$ at the field point $\mathbf{r}$ (in three dimensions), due to volume turbulence arising from a turbulent velocity field $v_m(\mathbf{r}_0)$ at the point $\mathbf{r}_0$ ($m = 1, 2, 3$), is given by Lighthill (1,2) as

$$p(\mathbf{r}) = \frac{\rho_0}{4\pi}\int \partial_{0m}\partial_{0n} v_m(\mathbf{r}_0) v_n(\mathbf{r}_0)\frac{e^{ik|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|}\, d\mathbf{r}_0 \tag{1}$$

where

$$\partial_{0m} \equiv \frac{\partial}{\partial X_{0m}}$$

differentiates on the "zero" coordinate. The Einstein summation convention is assumed here. The density of the medium is ρ_0 and $k = 2\pi/\lambda$. Further manipulations of this volume integral yield (1,2)

$$p(\mathbf{r}) = \frac{\rho_0}{4\pi}\partial_m\partial_n \int d\mathbf{r}_0 v_m(\mathbf{r}_0) v_n(\mathbf{r}_0)\frac{e^{ik|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|}\,. \tag{2}$$

In the far field (large r)

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|} \sim \frac{e^{ikr}}{r} e^{-ikr_0\cos\theta_0} \tag{3}$$

where $\mathbf{r} \cdot \mathbf{r}_0 = rr_0 \cos\theta_0$, and hence, for large r,

$$p(\mathbf{r}) \sim \left(\partial_m \partial_n \frac{e^{ikr}}{r}\right) Q_{mn} \tag{4}$$

where

$$Q_{mn} \equiv \frac{\rho_0}{4\pi} \int d\mathbf{r}_0 v_m(\mathbf{r}_0) v_n(\mathbf{r}_0) e^{-ikr_0 \cos\theta_0} . \tag{5}$$

If the turbulence is assumed to be isotropic,

$$v_m(\mathbf{r}) = v_m(r) , \tag{6}$$

and behaves like a spherical wave (with wavenumber K which could have a positive imaginar part), then

$$v_m(r) = v_m \frac{e^{iKr}}{Kr} . \tag{7}$$

Q_{mn} then can be evaluated as ($K = 2\pi/\Lambda$)

$$\begin{aligned} Q_{mn} &= \frac{\rho_0}{4\pi} \int_0^{\infty} dr_0 \int_0^{\pi} d\theta_0 \int_0^{2\pi} d\phi_0 r_0^2 \sin\theta_0 \, v_m(r_0) v_n(r_0) e^{-ikr_0 \cos\theta_0} \\ &= \frac{i\rho_0 v_m v_n}{2kK^2} \ln\left(\frac{1 + \frac{\Lambda}{2\lambda}}{1 - \frac{\Lambda}{2\lambda}}\right) \end{aligned} \tag{8}$$

where integral tables (3) have been used for the value of the r_0 integration. Introducing the adiabatic compressibility K_s, defined by

$$K_s = (\rho_0 c^2)^{-1}$$

where c is the velocity of sound in water, Eq. (8) becomes

$$Q_{mn} = \frac{iv_m v_n}{K_s c^2} \frac{1}{2kK^2} \ln\left(\frac{1 + \frac{\Lambda}{2\lambda}}{1 - \frac{\Lambda}{2\lambda}}\right).$$

Further approximation of this quadrupole term by its diagonal values yields (δ_{mn} is the Kronecker delta)

$$Q_{mn} \approx \frac{iM^2}{2K_s kK^2} \delta_{mn} \ln\left(\frac{1 + \frac{\Lambda}{2\lambda}}{1 - \frac{\lambda}{2\lambda}}\right) \tag{9}$$

where $M = v/c$ is the Mach number and $v = v_m$. In addition, in the far field, the term in brackets in Eq. (4) can be written as

$$\partial_m \partial_n \left(\frac{e^{ikr}}{r}\right) \sim (ik)^2 \left(\frac{e^{ikr}}{r}\right) \frac{\partial r}{\partial x_m} \frac{\partial r}{\partial x_n}$$

Using the approximation

$$\frac{\partial r}{\partial x_m} \frac{\partial r}{\partial x_n} \approx \delta_{mn}$$

with Eq. (9), and the result $\delta_{mn}\delta_{mn} = \delta_{nn} = 3$, it is possible to write Eq. (4) as

$$p(\mathbf{r}) \sim \frac{-3i}{2} \frac{e^{ikr}}{kr} \frac{M^2}{K_s} \left(\frac{\Lambda}{\lambda}\right)^2 \ln\left(\frac{1 + \frac{\Lambda}{2\lambda}}{1 - \frac{\Lambda}{2\lambda}}\right) \tag{10}$$

and the far-field intensity I as

$$I(r) \sim \frac{9}{8} \frac{cM^4}{(kr)^2 K_s} \left(\frac{\Lambda}{\lambda}\right)^4 \left[\ln\left(\frac{1 + \frac{\Lambda}{2\lambda}}{1 - \frac{\Lambda}{2\lambda}}\right)\right]^2 . \tag{11}$$

For very low frequencies (infrasound, λ large), Eq. (11) can be written

$$I(r) \sim \frac{9}{8} \frac{cM^4}{(kr)^2 K_s} \left(\frac{\Lambda}{\lambda}\right)^6 . \tag{12}$$

Hence, in addition to the usual spherical spreading loss, there is a dependence on the Mach number M raised to the fourth power, and to the ratio of turbulent wavelength to radiated wavelength raised to the sixth power. There is also a factor of λ in kr. Figure 1 is a plot of the ratio $I(r)/I_0$ versus Λ/λ from Eq. (12). The reference level is a comparison source I_0 having an output of 0.64×10^{-12} watts/cm^2 (a pressure level of 1 dyne/cm^2). The quantity I/I_0 is plotted in decibels at a range r of 1 naut mi and for an infrasonic wavelength $\lambda = 500$ ft (or a frequency of 10 Hz). The intensity at a new wavelength λ_1 can be found from the relation $I_1 = I(\lambda/\lambda_1)$. For example, doubling the wavelength ($\lambda_1 = 2\lambda$, frequency = 5 Hz) decreases each of the 10-Hz curves by 12 dB. Halving the wavelength (20 Hz) increases the curves by the same amount.

REMARKS

There are several assumptions and omissions in this calculation which should be restated. They are

a. Lighthill's turbulence theory for *free turbulence* is assumed valid, as is the far-field assumption. Effects due to boundary layer turbulence have been omitted.

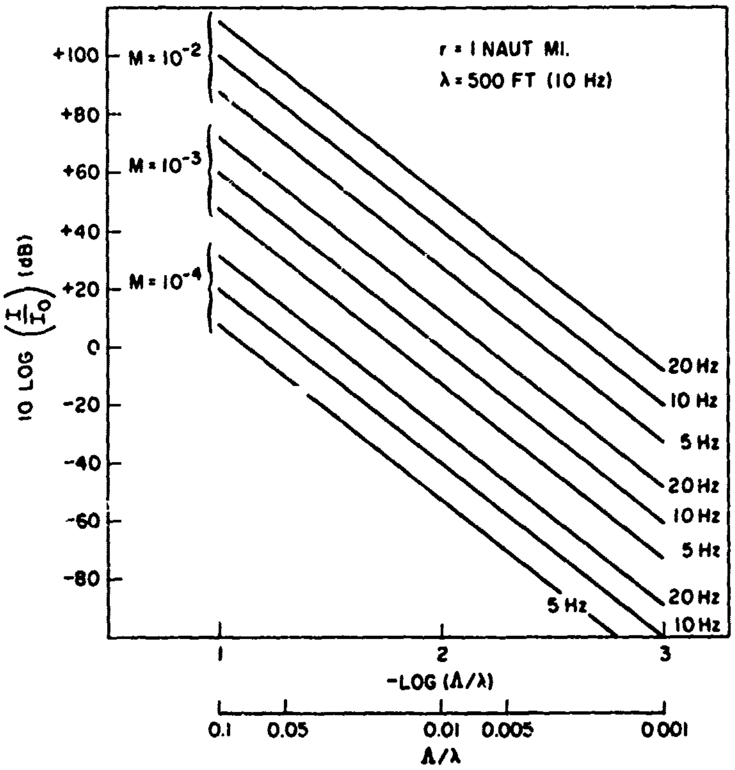

Fig. 1—Relative plot of radiation intensity I/I_0 vs the ratio of turbulent to radiated wavelength Λ/λ for several values of the Mach number M at a range r of 1 naut mi. For the corresponding values at 1 yd add 66 dB.

b. The turbulence was assumed to be *isotropic* and to radiate as a spherical wave without decay.

c. *No* attempt was made to treat the *time dependence* of the pressure or turbulent velocity fields.

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